CHAPTER 02

POLYNOMIALS

In the world of mathematics, polynomials are the bridge between simple arithmetic and complex equations.

INTRODUCTION

We have studied about polynomials in one variable, their factors and degrees in the previous class. We learnt about linear, quadratic and cubic polynomials. Also we got through monomial, binomial and trinomial etc. We studied about zeroes of a polynomial and factorization of algebraic expressions using various identities as well. Now we shall study about **geometrical interpretation of zeroes of a polynomial**. We shall learn how **to find zeroes of a quadratic polynomial** and we shall also establish **relationship between its coefficients and the zeroes**.

IMPORTANT TERMS & DEFINITIONS

01. Polynomial

An algebraic expression involving some constants and the variable terms is known as a polynomial.

If x be a variable, n be a positive integer and $a_0, a_1, a_2, ..., a_n$ be the constants (real numbers), then the expression of the form $\mathbf{a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0}$ is known as a polynomial in variable x with degree n (where $a_n \neq 0$).

A polynomial in x is generally denoted by $p(x), q(x), f(x), g(x)$ or $h(x)$.

◈ A polynomial of degree n can have at the most $(n+1)$ terms.

02. Degree of a polynomial

It is the exponent (power) of the highest degree term in a polynomial. That is, the highest power of variable x (*say*) in a polynomial is called its degree. For example, $2x^3 + 9x^2 - 7x + 5$ is a degree 3 polynomial.

03. Value of a polynomial

If $p(x)$ is a polynomial in x and $x = a$ is a real number, then the value of $p(x)$ which is obtained by putting $x = a$ in $p(x)$, is called the value of $p(x)$ at $x = a$ and it is denoted by $p(a)$.

04. Type of polynomials

(a) Constant polynomial

A polynomial of degree **zero** is called a constant polynomial. For example, $p(x) = a$ i.e., $p(x) = a\,x^0$, where a is any real number. For example, $p(x) = -5$, $f(x) = 2$.

(b) Linear polynomial

It is a polynomial of degree **one**. The general notation for a linear polynomial is given as $p(x) = ax + b$, $a \neq 0$, where a and b are constants. For example, $p(x) = 3x - 8$, $f(x) = x + \sqrt{2}$.

(c) Quadratic polynomial

A polynomial of degree **two** is called as quadratic polynomial. The general notation for a quadratic polynomial is given as $p(x) = ax^2 + bx + c,\ a \neq 0$, where a, b and c are constants.

For example, $p(x) = x^2 - 3x + 5$, $f(x) = 7x^2 + 4$.

◈ Just ponder if, $g(x) = 0x^2 + 7x + 4$ and $h(x) = x^2$ are quadratic polynomials or not.

(d) Cubic polynomial

A polynomial of degree **three** is called as cubic polynomial. The general notation for a cubic polynomial is given as $p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, where a, b, c and d are constants. For example, $p(x) = x^3 - 3x^2 + 5$, $f(x) = 4x^3 + 7x^2 - 2x + 4$.

(e) Bi-quadratic polynomial

It is a polynomial of degree **four**. The general notation for a bi-quadratic polynomial is given as $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, $a \neq 0$, where a, b, c, d and e are constants.

For example, $p(x) = x^4 - 2x^3 + 3x^2 + 5$, $f(x) = 6x^4 + 4x^3 - 7x^2 - 2x + 3$.

Also a polynomial of degree *four* with *integral coefficients* is called a quartic polynomial.

05. Zeroes of a polynomial

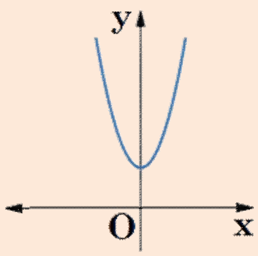
The values of x for which the polynomial $p(x)$ becomes zero are called the zeroes of the polynomial.

In other words, if $p(x)$ is a polynomial, then the zeroes of polynomial $p(x)$ are the solutions to the equation $p(x) = 0$. Mathematically, if $p(a) = 0$, then 'a' is a zero of the polynomial $p(x)$.

- **a.** A polynomial of degree n has at the most n **real** zeroes. Also, the polynomial of degree n has exactly n zeroes (real or imaginary zeroes).
- **b.** A quadratic polynomial has at the most two **real** zeroes. Also, it has exactly two zeroes (real or imaginary zeroes).
- **c.** Geometrically, zeroes of a polynomial $p(x)$ are the x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis. Further, if a polynomial is of the form $x = p(y)$, then geometrically, its zeroes are y-coordinates of the points where the graph of $x = p(y)$ intersects the y-axis.

Let's draw clarity about above discussions.

Take a polynomial $p(x) = x^2 + 9$. We know its degree is 2, so it can have **at most 2 real zeroes**. We do **not** have any real values of x, for which $p(x) = x^2 + 9 = 0$. That is, for $p(x) = x^2 + 9$ we have **no real zeroes**; that's why the graph of polynomial $p(x) = x^2 + 9$ does not intersect x-axis at any point. Refer the figure.



However $p(x) = x^2 + 9$ surely has 2 zeroes which are $x = \pm 3i$, we shall study about **imaginary zeroes** in higher classes.

*Therefore in class X, we conclude that $p(x) = x^2 + 9$ has **no zeroes** (keep in your mind that, we have not studied about the imaginary zeroes yet).*

06. Relationship between zeroes of a Quadratic polynomial and Coefficients

If α and β are the zeroes of quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$, then

$$\textbf{Sum of Zeroes } (\alpha + \beta) = -\frac{b}{a} \textbf{ and, Product of Zeroes } (\alpha\beta) = \frac{c}{a}.$$

$$\text{i.e., } \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \text{ and, } \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

***07. Relationship between zeroes of a Cubic polynomial and Coefficients**

If α, β and γ are the zeroes of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, then

$$\alpha + \beta + \gamma = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \text{ and, } \alpha\beta\gamma = -\frac{d}{a}.$$

08. To find a Quadratic polynomial if its zeroes are given

If α and β are the zeroes of quadratic polynomial *say* $p(x)$, then the polynomial is given as,

$$\mathbf{p(x) = x^2 - Sx + P \;\; or, \;\; k(x^2 - Sx + P)}$$

where $S = \alpha + \beta,\ P = \alpha\beta$ and k is any non-zero real number.

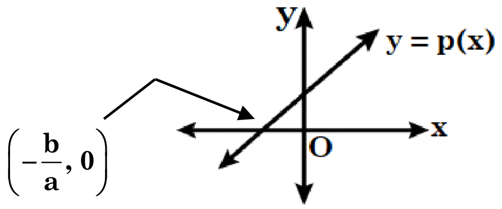
◈ Note that here S and P represent the sum of zeroes and product of zeroes of $p(x)$ respectively.

09. Graph of polynomials

(a) Linear polynomial

For a linear polynomial $p(x) = ax + b,\ a \neq 0$, the graph is a straight line and it intersects x-axis at exactly one point $\left(-\frac{b}{a}, 0\right)$. The zero of polynomial $p(x)$ is $x = -\frac{b}{a}$.

To make the graph, we take $y = p(x)$ i.e., $y = ax + b$.

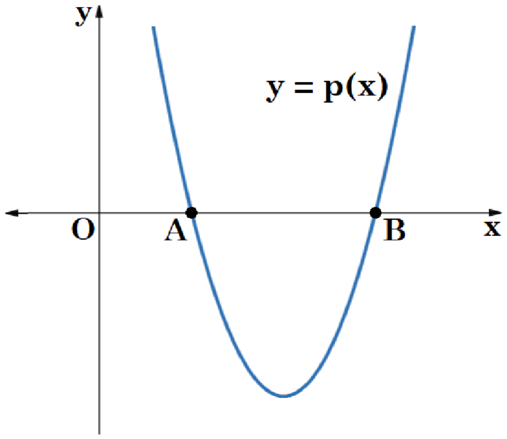


(b) Quadratic polynomial

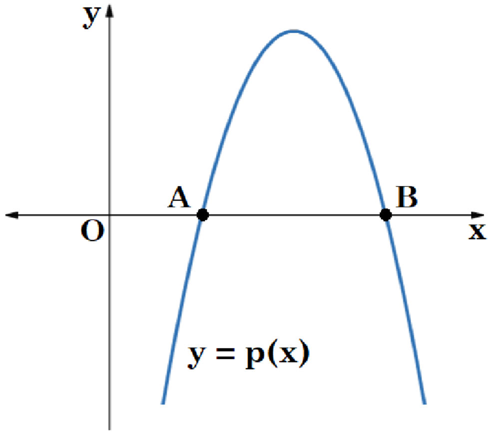
For a quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$, the graph is a parabola; which is open either upwards ($\cup$) or downwards ($\cap$).

If $a > 0$, then shape is upward and; if $a < 0$, then the shape is downward.

Case I. In both the graphs of quadratic polynomial shown below, the curves cut x-axis at two distinct points A and B. Therefore, the quadratic polynomial has **two zeroes**. The x-coordinates of points A and B are the two zeroes of the polynomial.

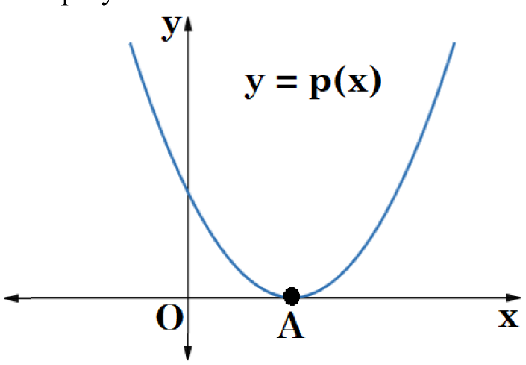


$(a > 0,\ b^2 - 4ac > 0)$

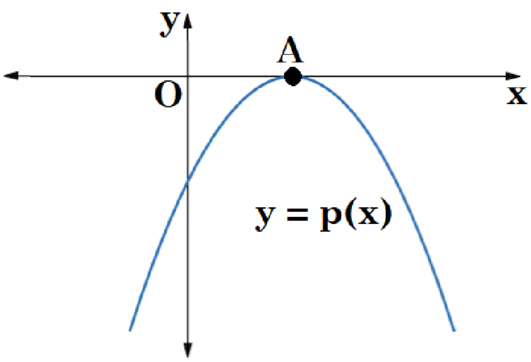


$(a < 0,\ b^2 - 4ac > 0)$

Case II. In both the graphs of quadratic polynomial shown below, the curves cut x-axis at **exactly one point** A i.e., at **two coincident points**. The two points A and B in **Case I** coincide here to become one point A. Therefore, the quadratic polynomial has **only one zero**. The x-coordinate of point A is the **only one zero** of the polynomial.

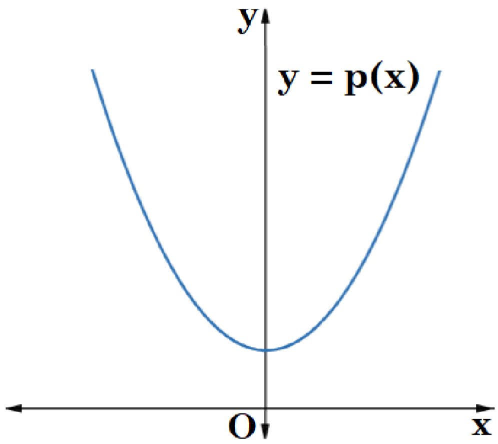


$(a > 0,\ b^2 - 4ac = 0)$

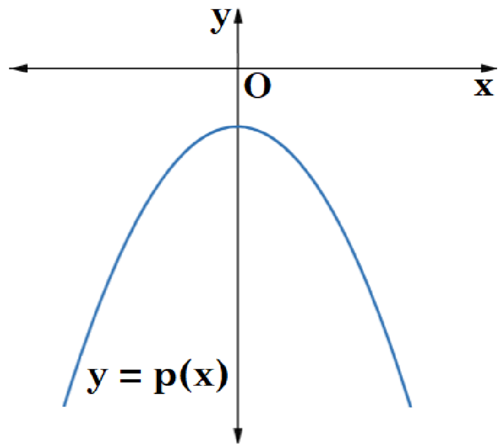


$(a < 0, b^2 - 4ac = 0)$

Case III. In both the graphs of quadratic polynomial shown below, the curves are either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at **any point**. Therefore, the quadratic polynomial has **no zero**.



$(a > 0,\ b^2 - 4ac < 0)$



$(a < 0, b^2 - 4ac < 0)$

Note that, to make the graph we take $y = p(x)$ i.e., $y = ax^2 + bx + c$.

<u>**Remark**</u> The graph of quadratic polynomial intersects x-axis at maximum two points. Geometrically a quadratic polynomial can have either **two distinct zeroes** (when $b^2 - 4ac > 0$) **or two equal zeroes i.e., one zero** (when $b^2 - 4ac = 0$)**, or no zero** (when $b^2 - 4ac < 0$). This also means that a polynomial of degree 2 has **at most two zeroes**.

> *It is important to note that the concept of **imaginary zeroes is not taught in class X**. So, in case of declaring **zeroes of a polynomial of degree n**, we usually say that we have **at most n zeroes** (which should be otherwise, **exactly n zeroes** or, **at most n real zeros** in higher classes).*

10. Algebraic Identities

(a) $(a+b)^2 = a^2 + 2ab + b^2$

(b) $(a-b)^2 = a^2 - 2ab + b^2$

(c) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(d) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(e) $a^2 - b^2 = (a+b)(a-b)$

(f) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(g) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(h) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. Find the zeroes of $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify the relationship between its zeroes and coefficients.

Sol. We have, $\sqrt{3}x^2 - 8x + 4\sqrt{3}$

$\Rightarrow \quad = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$\Rightarrow \quad = \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$\Rightarrow \quad = (x - 2\sqrt{3})(\sqrt{3}x - 2)$

So, the value of $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ is zero when $(x - 2\sqrt{3}) = 0$ or $(\sqrt{3}x - 2) = 0$ i.e., when

$x = 2\sqrt{3}$ or $x = \dfrac{2}{\sqrt{3}}$.

Therefore, the zeroes of $\sqrt{3}x^2-8x+4\sqrt{3}$ are $x=2\sqrt{3}$ and $x=\dfrac{2}{\sqrt{3}}$.

Now, sum of zeroes $=2\sqrt{3}+\dfrac{2}{\sqrt{3}}=\dfrac{8}{\sqrt{3}}=-\left(-\dfrac{8}{\sqrt{3}}\right)=-\dfrac{\text{Coefficient of x}}{\text{Coefficient of }x^2}$

and, product of zeroes $=(2\sqrt{3})\times\left(\dfrac{2}{\sqrt{3}}\right)=4=\dfrac{4\sqrt{3}}{\sqrt{3}}=\dfrac{\text{Constant term}}{\text{Coefficient of }x^2}$.

Ex02. Find the zeroes of quadratic polynomial $p(x)=2x^2-7x-15$ and verify the relationship between its coefficients and zeroes.

Sol. We have, $p(x)=2x^2-7x-15$

$\Rightarrow p(x)=2x^2-10x+3x-15$

$\Rightarrow p(x)=2x(x-5)+3(x-5)$

$\Rightarrow p(x)=(x-5)(2x+3)$

For $p(x)=0$, we have either $(x-5)=0$ or $(2x+3)=0$

That is, either $x=5$ or $x=-\dfrac{3}{2}$.

Therefore, the zeroes of $p(x)=2x^2-7x-15$ are $x=5$ and $x=-\dfrac{3}{2}$.

Now, sum of zeroes $=5+\left(-\dfrac{3}{2}\right)=\dfrac{10-3}{2}=\dfrac{7}{2}=-\left(-\dfrac{7}{2}\right)=-\dfrac{\text{Coefficient of x}}{\text{Coefficient of }x^2}$

and, product of zeroes $=(5)\times\left(-\dfrac{3}{2}\right)=\dfrac{-15}{2}=\dfrac{\text{Constant term}}{\text{Coefficient of }x^2}$.

Ex03. Find a quadratic polynomial whose zeroes are $\dfrac{1}{4}$ and -1.

Sol. Let the required quadratic polynomial be $p(x)=ax^2+bx+c,\ a\neq 0$, and its zeroes be denoted by α and β.

We have, sum of zeroes $=S=\alpha+\beta=\dfrac{1}{4}+(-1)=-\dfrac{3}{4}$

and, product of zeroes $=P=\alpha\beta=\dfrac{1}{4}\times(-1)=-\dfrac{1}{4}$.

So, the required quadratic polynomial is $p(x)=x^2-Sx+P$ or $k(x^2-Sx+P)$, where $k\neq 0$.

$\therefore\ p(x)=x^2-\left(-\dfrac{3}{4}\right)x+\left(-\dfrac{1}{4}\right)=x^2+\dfrac{3}{4}x-\dfrac{1}{4}$ or $k(4x^2+3x-1)$ where k is any non-zero real no.

Ex04. Write a quadratic polynomial whose zeroes are reciprocal of the zeroes of the quadratic polynomial $p(x)=a\,x^2+b\,x+c,\ a\neq 0,\ c\neq 0$.

Sol. Let α and β be the zeroes of $p(x)=a\,x^2+b\,x+c$.

Then $\alpha+\beta=-\dfrac{b}{a},\ \alpha\beta=\dfrac{c}{a}$.

For the required quadratic polynomial, the zeroes will be $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

So, for the required polynomial, $S=\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\alpha+\beta}{\alpha\beta}=\dfrac{\frac{-b}{a}}{\frac{c}{a}}=-\dfrac{b}{c}$; $P=\dfrac{1}{\alpha}\times\dfrac{1}{\beta}=\dfrac{1}{\alpha\beta}=\dfrac{1}{\frac{c}{a}}=\dfrac{a}{c}$.

Therefore, the required polynomial is $x^2 - Sx + P$ or $k(x^2 - Sx + P)$, where $k \neq 0$

That is, $x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}$ or $k\left[x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}\right]$, where $k \neq 0$

Hence, the polynomial is $x^2 + \frac{b}{c}x + \frac{a}{c}$ or $\frac{k}{c}\left[cx^2 + bx + a\right]$, where $k \neq 0$.

$\therefore$ The required polynomial is $x^2 + \frac{b}{c}x + \frac{a}{c}$ or $\lambda\left[cx^2 + bx + a\right]$, where $\lambda = \frac{k}{c}$; $\lambda \neq 0$.

Ex05. If α and β are the zeroes of quadratic polynomial $x^2 + 3x + 2$, then find a quadratic polynomial whose zeroes are $\alpha + 1$ and $\beta + 1$.

Sol. For the ploynomial $x^2 + 3x + 2$, $\alpha + \beta = -3$ and $\alpha\beta = 2$.

The sum of zeroes of required quadratic polynomial whose zeroes are $\alpha + 1$ and $\beta + 1$

$$= (\alpha+1) + (\beta+1) = \alpha + \beta + 2 = -3 + 2 = -1$$

Also, the product of zeroes of required quadratic polynomial

$$= (\alpha+1).(\beta+1) = \alpha\beta + (\alpha+\beta) + 1 = 2 + (-3) + 1 = 0$$

Hence, the required polynomial is $x^2 - (-1)x + 0$ i.e., $x^2 + x$ or, $k(x^2 + x)$; $k \neq 0$.

Ex06. If α and β are the zeroes of quadratic polynomial $x^2 - ax - b$, then find a quadratic polynomial whose zeroes are $3\alpha + 1$ and $3\beta + 1$.

Sol. For the ploynomial $x^2 - ax - b$, $\alpha + \beta = -\left(\frac{-a}{1}\right) = a$ and $\alpha\beta = \frac{-b}{1} = -b$.

The sum of zeroes of required quadratic polynomial whose zeroes are $3\alpha + 1$ and $3\beta + 1$

$$= (3\alpha+1) + (3\beta+1) = 3(\alpha+\beta) + 2 = 3a + 2$$

Also, the product of zeroes of required quadratic polynomial

$$= (3\alpha+1).(3\beta+1) = 9(\alpha\beta) + 3(\alpha+\beta) + 1 = -9b + 3a + 1$$

Hence, the required polynomial is given as $x^2 - (3a+2)x + (3a - 9b + 1)$

Note that, the required polynomial can also be taken as $k\left[x^2 - (3a+2)x + (3a - 9b + 1)\right]$; $k \neq 0$.

Ex07. If α and β are the zeroes of the polynomial $p(x) = x^2 - 3x - 1$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. For $p(x) = x^2 - 3x - 1$, we have $\alpha + \beta = -\left(\frac{-3}{1}\right) = 3$, $\alpha\beta = \frac{-1}{1} = -1$.

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{-1} = -3$.

Ex08. α and β are the zeroes of the polynomial $5x^2 - 16x - 10$. Find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Sol. For $5x^2 - 16x - 10$, we have $\alpha + \beta = -\left(\frac{-16}{5}\right) = \frac{16}{5}$, $\alpha\beta = \frac{-10}{5} = -2$.

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\left(\frac{16}{5}\right)^2 - 2(-2)}{-2} = \frac{\frac{256}{25} + 4}{-2} = -\frac{128}{25} - 2 = -\frac{128 + 50}{25} = -\frac{178}{25}.$$

Ex09. If α and β are the zeroes of $f(y) = y^2 - 5y + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Sol. For $f(y)=y^2-5y+3$, we have $\alpha+\beta=-\left(\frac{-5}{1}\right)=5$, $\alpha\beta=\frac{3}{1}=3$.

Now $\alpha^4\beta^3+\alpha^3\beta^4=\alpha^3\beta^3(\alpha+\beta)=(\alpha\beta)^3(\alpha+\beta)=(3)^3(5)=27\times 5$

$\therefore \alpha^4\beta^3+\alpha^3\beta^4=135$.

Ex10. If p and q are zeroes of the polynomial $p(y)=21y^2-y-2$, then find the value of $(1-p)(1-q)$.

Sol. For the polynomial $p(y)=21y^2-y-2$, we have $p+q=-\left(\frac{-1}{21}\right)=\frac{1}{21}$, $pq=\frac{-2}{21}$.

Now $(1-p)(1-q)=1-p-q+pq=1-(p+q)+pq$

$$=1-\frac{1}{21}+\left(-\frac{2}{21}\right)=\frac{21-1-2}{21}=\frac{18}{21}=\frac{6}{7}.$$

Ex11. If the zeroes of the polynomial x^2+ax+b are in the ratio $3:4$, then prove that $12a^2=49b$.

Sol. Assume that the zeroes of x^2+ax+b are 3α and 4α.

Then the sum of the zeroes is $3\alpha+4\alpha=-\left(\frac{a}{1}\right)$ $\Rightarrow 7\alpha=-a$ ...(i)

Also the product of the zeroes is $3\alpha\times 4\alpha=\frac{b}{1}$ $\Rightarrow 12\alpha^2=b$ ...(ii)

By (i) and (ii), we get $12\left(-\frac{a}{7}\right)^2=b$

$\Rightarrow 12\times\frac{a^2}{49}=b$

$\therefore 12a^2=49b$.

Ex12. If one zero of the polynomial $p(x)=6x^2+37x-k+2$ is reciprocal of the other, find the value of k.

Sol. Let α and β be the zeroes of $p(x)=6x^2+37x+(-k+2)$ such that $\alpha=\frac{1}{\beta}$.

Clearly, $\alpha\beta=1$ ...(i)

Now taking the 'product of zeroes' of polynomial, we get $\alpha\beta=\frac{-k+2}{6}$

By (i), $\alpha\beta=\frac{-k+2}{6}=1$

$\Rightarrow -k+2=6$

$\Rightarrow -k=6-2$

$\therefore k=-4$.

Ex13. Find the value of 'k' such that the polynomial $p(x)=3x^2+2kx+x-k-5$ has the sum of zeroes equal to half of their product.

Sol. Rewriting the polynomial, we get $p(x)=3x^2+(2k+1)x+(-k-5)$.

So, for the polynomial $p(x)$, the sum of zeroes is $-\frac{2k+1}{3}$ and the product of zeroes is $-\frac{k+5}{3}$.

Now according to the given condition, $-\frac{2k+1}{3}=\frac{1}{2}\times\left(-\frac{k+5}{3}\right)$ i.e., $2k+1=\frac{k+5}{2}$

$\Rightarrow 4k+2=k+5$ $\Rightarrow 4k-k=5-2$

$\therefore k=1$.

Ex14. Find the zeroes of $p(x)=(x+4)^2-(x-4)^2$.

Sol. We have $p(x)=(x+4)^2-(x-4)^2$

$\Rightarrow p(x)=(x^2+8x+16)-(x^2-8x+16)$

$\Rightarrow p(x)=x^2+8x+16-x^2+8x-16$

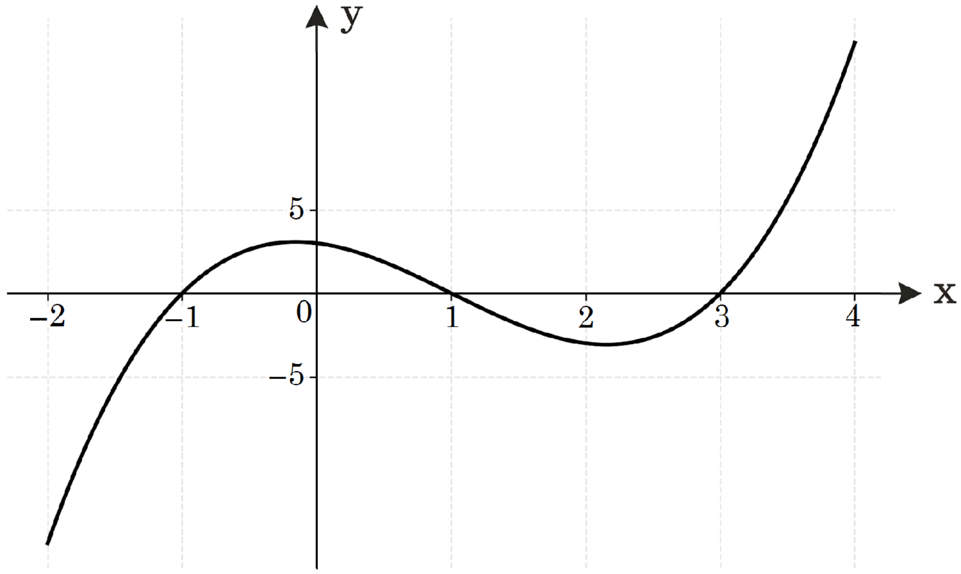
$\Rightarrow p(x)=16x$

For $p(x)=0$, we have $16x=0$

$\therefore x=0$.

Therefore, the zero of given polynomial $p(x)$ is 0.

Ex15. The graph of polynomial p(x) is shown in figure. Write the number of zeroes.

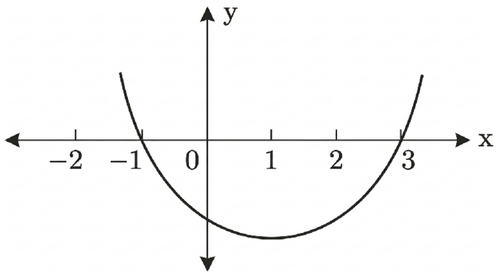


Sol. To find the number of zeroes of a polynomial from its graph, we look at the number of times the graph intersects the x-axis. Each point where the graph touches or crosses the x-axis represents a zero of the polynomial.

The given curve crosses the x-axis at 3 distinct points.

Hence, the number of zeroes of the polynomial p(x) is 3.

Ex16. The graph of a quadratic polynomial $p(x)$ is shown in the figure.



Answer the questions on the basis of shown graph.

(i) How many zeroes polynomial $p(x)$ has?

(ii) Find the zeroes of $p(x)$.

(iii) Write the sum and product of zeroes of $p(x)$.

(iv) Write the polynomial $p(x)$.

Sol. The graph is of a quadratic polynomial.

(i) Since the zeroes of a polynomial are the x-coordinates where the graph intersects the x-axis.

Looking at the graph, we observe that graph of $p(x)$ cuts x-axis at two points.

So, $p(x)$ has two zeroes.

(ii) The first intersection is at $x=-1$ and the second intersection is at $x=3$.

So, the zeroes are –1 and 3.

(iii) Sum of the zeroes $=(-1)+3=2$; and the Product of the zeroes $=(-1)(3)=-3$.

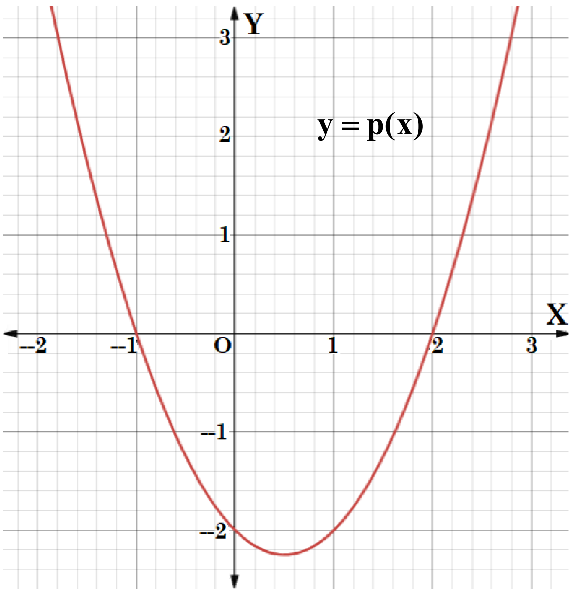
(iv) $\because$ Sum of the zeroes $(S)=2$; and the Product of the zeroes $(P)=-3$.

So, the required quadratic polynomial $p(x)$ is x^2-Sx+P or $k(x^2-Sx+P)$, where $k\neq 0$

That is, x^2-2x-3 or $k(x^2-2x-3)$, where $k\neq 0$.

Ex17. Consider the graph of a polynomial $p(x)$.

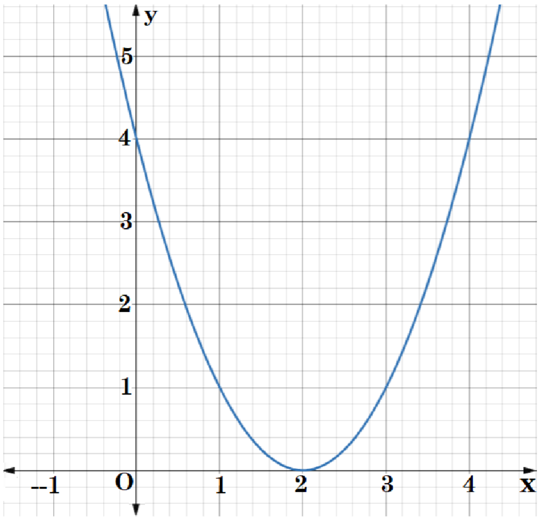
(i) Find how many zeroes it can have.
(ii) Write the zeroes of polynomial $p(x)$ if possible.
(iii) Write the polynomial $p(x)$.



Sol. (i) Since the graph of polynomial cuts x-axis at two points. Therefore, it has two zeroes.
(ii) The points at which the graph cuts x-axis are (–1, 0) and (2, 0).
The corresponding x-coordinates are –1 and 2.
Therefore, the zeroes of $p(x)$ are -1 and 2.
(iii) $p(x) = x^2 - (-1+2)x + (-1)(2)$
That is, $p(x) = x^2 - x - 2$ or $k(x^2 - x - 2)$; where k is any non-zero real constant.

Ex18. The graph of polynomial $p(x) = (x-2)^2$ is shown in the figure given below.

(i) Write the degree of polynomial p(x).
(ii) Write the number of zeroes of p(x), as per the degree of given polynomial.
(iii) Analyzing the graph shown, how many zeroes are there for p(x)? Write the zeroes.



Sol. (i) Degree : 2, as the highest exponent of variable x in $p(x) = (x-2)^2 = x^2 - 4x + 4$ is 2.
(ii) As the degree of p(x) is 2 so, there are **exactly two zeroes**.
However the **polynomial will have repeated zeroes (i.e., two equal zeroes)**, as there is a multiplicity of 2. Note that in polynomial $p(x) = (x-2)^2$, the factor $(x-2)$ appears twice, so there is multiplicity of 2 for the polynomial.
(iii) Note that the polynomial touches the x-axis at only **one** point i.e., at $x = 2$. So, the polynomial $p(x)$ has Single (**only one**) real zeroes. Also, the zero of polynomial $p(x)$ is 2.

Exercise 2.1

Q01. Which of the followings are **not** polynomials?

(a) $3x^3 + x^2 + x^{-2} + 7$ (b) $x^2 + px + q$

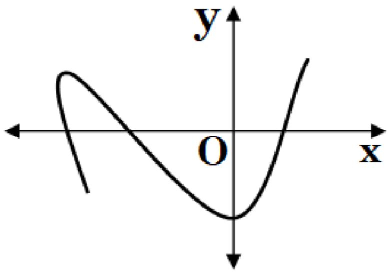
(c) $x^2 + \frac{1}{x^2} + 8$ (d) $2x^3 + 3x^2 - 5x - 6$.

Q02. What do you understand by the value of a polynomial at a given point?

Q03. If $p(x) = 3x^3 - 2x^2 + 6x - 5$, find p(2).

Q04. Given a linear polynomial in x, state how many zeroes it can have and why? Illustrate with the help of an example.

Q05. The graph of polynomial p(x) is shown in figure.



Write the number of zeroes of p(x).

Q06. Is $7x^2 - \sqrt{x} + 2$ a polynomial? If yes, write its degree. Justify your answer.

Q07. Write a quadratic polynomial whose zeroes are given as α and β.

Exercise 2.2

Q01. Find the zeroes of the polynomial $mx^2 + (m+n)x + n$.

Q02. Show that the quadratic polynomial $x^2 + 4x + 5$ has no real zeroes.

Q03. Find the zeroes of $x^2 - 2$ and verify the relationship between the zeroes and the coefficients.

Q04. Find the zeroes of $f(m) = 4m^2 + 8m$ and hence verify the relationship between the zeroes and its coefficient.

Q05. Find the zeroes of the quadratic polynomial $f(x) = 6x^2 - x - 15$ and establish a relationship between the zeroes and its coefficients.

Q06. Find the zeroes of $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ and verify the relationship between the zeroes and its coefficients.

Q07. Find the zeroes of the polynomial $2x^2 - 7x + 3$ and hence find the sum and product of its zeroes.

Q08. Find the sum and product of the zeroes of $p^2x^2 + (p^2 - q^2)x - q^2$, also cite the relationship between the zeroes and the coefficients.

Exercise 2.3

Q01. Find a quadratic polynomial whose zeroes are – 5 and 7.

Q02. Form a quadratic polynomial whose sum and product of zeroes are $\sqrt{2}$ and $\frac{1}{3}$ respectively.

Q03. Find a quadratic polynomial whose sum and product of zeroes are $\sqrt{2}$ and 3 respectively.

Q04. Find the quadratic polynomial whose one zero is $2 + \sqrt{3}$.

Q05. Form a quadratic polynomial whose zeroes are 2 and –6. Verify the relation between the coefficients and zeroes of the polynomial.

Q06. If a and b are zeroes of the polynomial $x^2 - x - 6$, then find a quadratic polynomial whose zeroes are $3a + 2b$ and $2a + 3b$.

Q07. Form a quadratic polynomial whose zeroes are the squares of the zeroes of $x^2 - 2x - 5$.

Q08. Write the zeroes of the polynomial $p(x) = x^2 - \sqrt{3}x - \sqrt{2}x + \sqrt{6}$. Also write another polynomial whose zeroes are square of the zeroes of $p(x)$.

Exercise 2.4

Q01. If 1 is a zero of the polynomial $f(u) = au^2 - 3(a-1)u - 1$, then find the value of a.

Q02. For what value of k, –4 is a zero of the polynomial $x^2 - x - (2k+2)$?

Q03. If one zero of the quadratic polynomial $x^2 + 3kx + 8k$ is 2, then find the value of k and the other zero of the polynomial.

Q04. If one zero of polynomial $(k^2+9)x^2 + 13x + 6k$ is reciprocal of the other, find the value of k.

Q05. For what value of k, the polynomial $p(x) = (k^2 - 4k + 4)x^2 + (k+3)x + (2k-4)$ is quadratic? If one zero of the polynomial $p(x)$ is reciprocal of the other, then find the value/s of k.

Q06. If one zero of $4x^2 - 9 - 8kx$ is negative of the other, determine the value of k.

Q07. If 1 is a zero of the polynomials $at^2 + at + 3$ and $t^2 + t + b$, then find the value of (ab).

Q08. The sum of the zeroes of the quadratic polynomial $f(y) = ky^2 + 2y + 3k$ is same as their product, determine the value of k.

Q09. If sum of the zeroes of $kx^2 + 3k + 2x$ is equal to their product, then write the value of k.

Q10. Find the value of k such that the polynomial $3x^2 + 2kx + x - k - 5$ has the sum of its zeroes as half of their product.

Q11. If –5 is one of the zeroes of $2x^2 + px - 15$ and quadratic polynomial $p(x^2 + x) + m$ has both of its zeroes equal to each other, then find the value of m.

Q12. The difference between the squares of the zeroes of $x^2 + px + 45$ is 216, find the value of p.

Q13. If α and β are zeroes of $2x^2 - 7x + 3$, then find the sum of reciprocal of the square of its zeroes.

Q14. If the zeroes of $x^2 + (a+1)x + b$ are 2 and –3, then find the values of a and b.

Exercise 2.5

Q01. If α and β are the zeroes of polynomial $ax^2 + bx + c,\ a \neq 0$, then evaluate:

(a) $\alpha^2 + \beta^2$ (b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(c) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2$ (d) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$

Q02. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Q03. If sum of the squares of zeroes of $f(x) = x^2 - 8x - \lambda$ is 40, then find the value of λ.

Q04. If α and β are the zeroes of polynomial $f(x) = 2x^2 + 5x + m$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of m.

Q05. If α and β are the zeroes of polynomial $f(x) = x^2 - x - 2$, then find a polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

Q06. If α and β are the zeroes of polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, then find the value of k.

Q07. If α and β are the zeroes of $2x^2 + 7x - 3$, then find the sum of the reciprocal of its zeroes.

Q08. If m and n are zeroes of $3x^2 + 11x - 4$, then find the value of $\frac{m}{n} + \frac{n}{m}$.

Q09. If p and q are zeroes of polynomial $t^2 - 4t + 3$, then show that $\frac{1}{p} + \frac{1}{q} - 2pq + \frac{14}{3} = 0$.

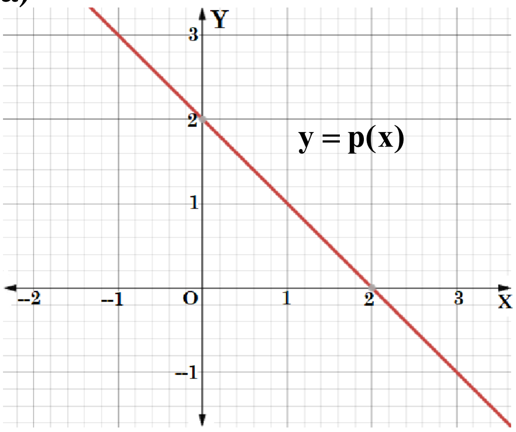
Q10. If $(x-6)$ is a factor of $x^3 + ax^2 + bx = 0$ and $a - b = 7$, then find the values of a and b.

Q11. For what value of k, the polynomial $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by $x + 7$.

Q12. The coefficient of x in the quadratic polynomial $f(x) = x^2 + px + q$ was wrongly written as 17 in place of 13 and the zeroes thus found were –2 and –15. Find the zeroes of correct polynomial.

Q13. Find the number of zeroes and values of zeroes (if possible) in the following graphs.

(a)



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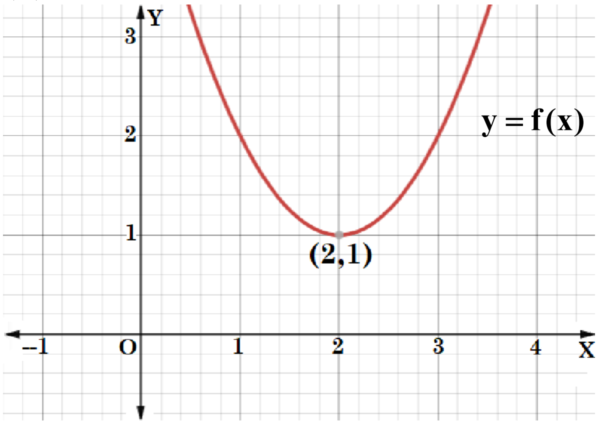

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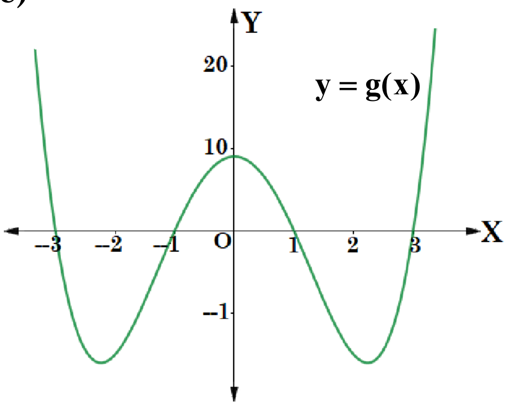

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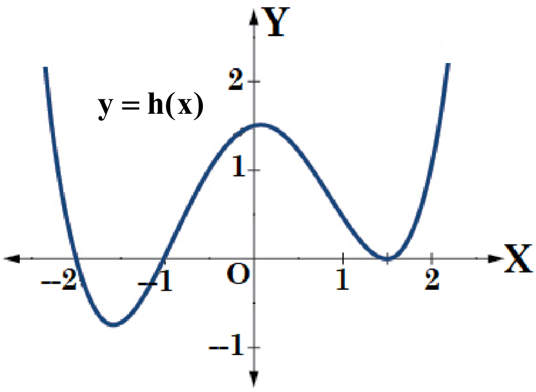

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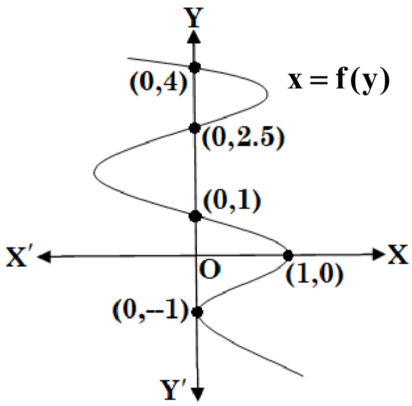
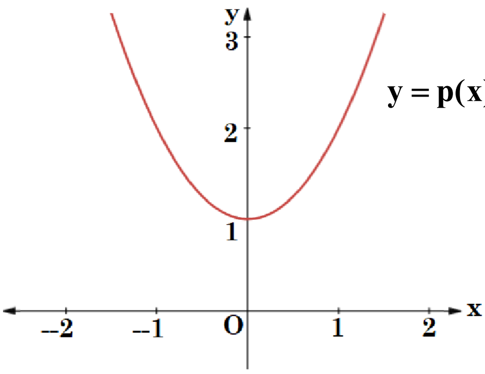
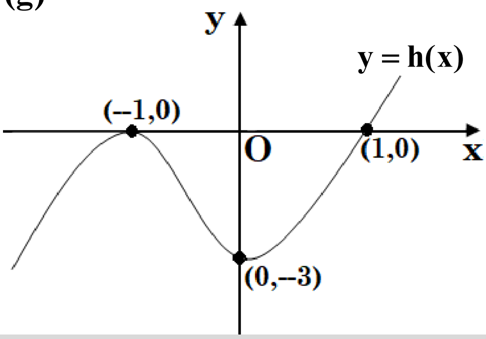


(f)



(g)

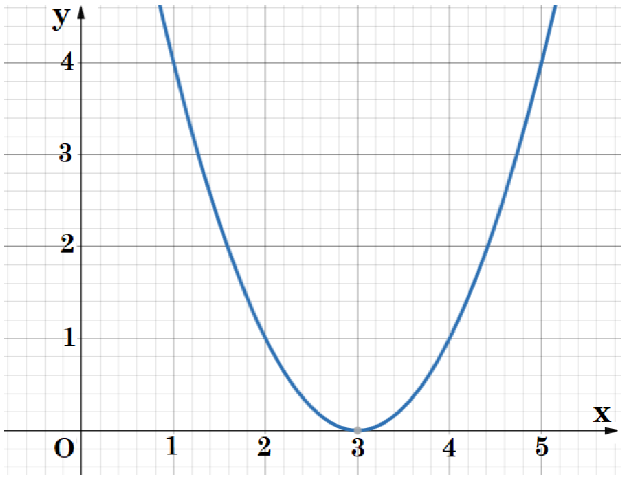


Q14. If α and β are the zeroes of the polynomial $f(x) = ax^2 + bx + c$, then find $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$.

Q15. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax + b$, then find the value of $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} - \frac{a^4}{b^2} + \frac{4a^2}{b}$.

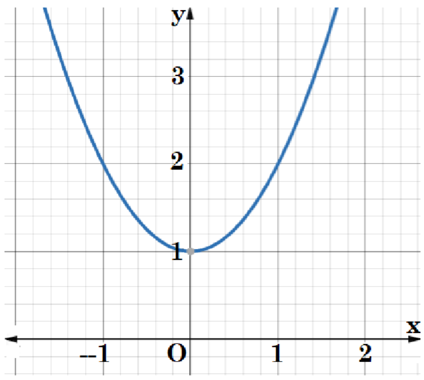
Q16. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 4x + 1$, then find a quadratic polynomial whose zeores are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Q17. The graph of polynomial $p(x) = (x-3)^2$ is shown in figure.



(i) Write the degree of polynomial p(x).
(ii) Write the number of zeroes of polynomial p(x), as per the degree.
(iii) Analyzing the graph shown above, how many zeroes are there for p(x), write the zeroes.

Q18. The graph of polynomial $p(x) = x^2 + 1$ is shown in figure.



(i) Write the degree of polynomial p(x).
(ii) Write the number of zeroes of polynomial p(x), as per the degree.
(iii) Analyzing the graph shown above, how many real zeroes are there for p(x)?

A more general insight for no. of zeroes of polynomials (no. of roots of equations) **can be read by Scanning the QR-Code** given here!

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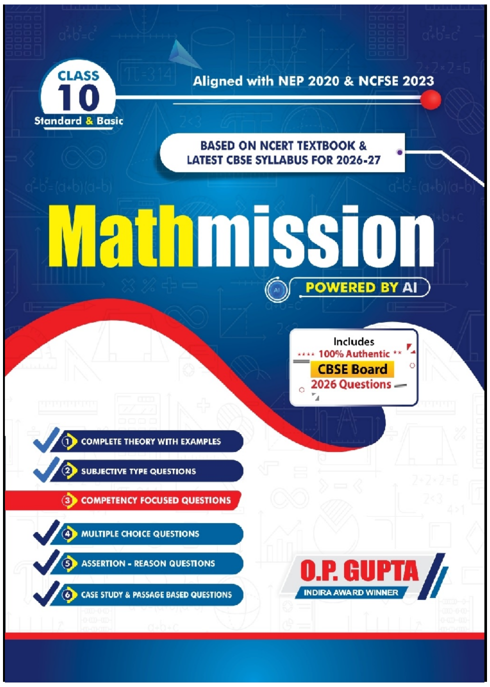

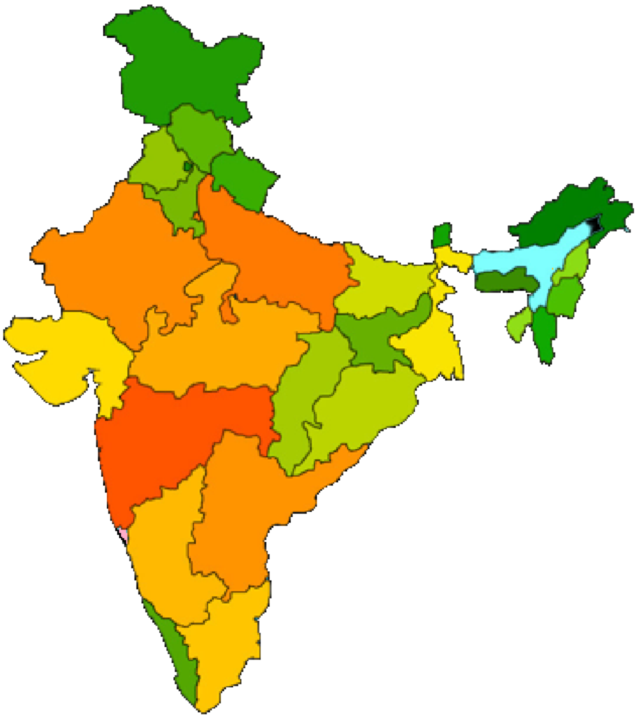

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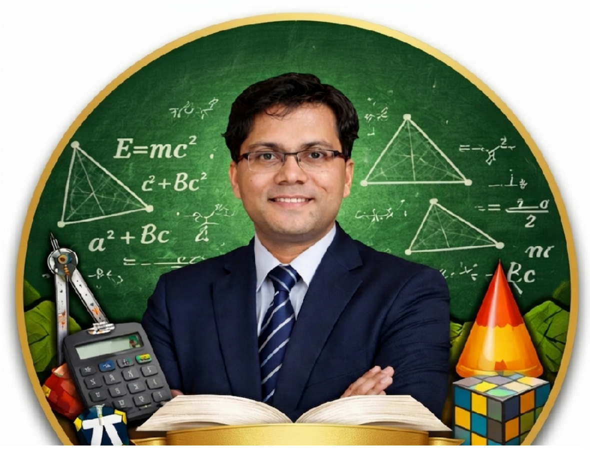

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O.P. GUPTA having taught math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn math without being afraid. Undoubtedly his mathematics books are best sellers on amazon and Flipkart.

His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

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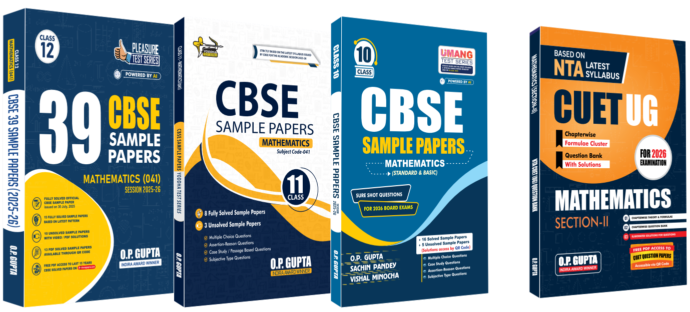

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